Quiz 9: November 23, 2021

Name: ____________________

Student ID: ____________________

1. (10 points) if $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2$, with $a_0 = 2, a_1 = 1$, find the closed form of this Linear Homogeneous Recurrence Relation of order 2 with constant coefficients.

   Make sure you give your characteristic equation (3 points), find your roots (2 points), write down the solution in terms of unknown constants (2 points), and then solve the two equations with two unknowns to come up with a final solution (3 points).

$$\left.\begin{array}{l} a_n = 6a_{n-1} - 8a_{n-2} \\ * \ \boxed{a_n = C_1 a_{n-1} + C_2 a_{n-2}} \end{array}\right\} \Rightarrow \begin{array}{l} C_1 = 6 \\ C_2 = -8 \end{array}$$

$$* \ \boxed{r^2 - C_1 r - C_2 = 0}$$

$$r^2 - 6r + 8 = 0 \Rightarrow (r-4)(r-2) = 0$$

$$r_1 = 4 \qquad r_2 = 2$$

$$* \ \boxed{a_n = \alpha_1 r_1^n + \alpha_2 r_2^n} \rightarrow \boxed{a_n = \frac{-3}{2} \cdot 4^n + \frac{7}{2} \cdot 2^n}$$

closed form

$$a_n = \alpha_1 . 4^n + \alpha_2 \cdot 2^n$$

$$\text{initials} \left\{ \begin{array}{l} a_{n=0} = 2 = \alpha_1 . 4^0 + \alpha_2 \cdot 2^0 \\ a_{n=1} = 1 = \alpha_1 . 4^1 + \alpha_2 . 2^1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \alpha_1 + \alpha_2 = 2 \\ 4\alpha_1 + 2\alpha_2 = 1 \end{array} \right.$$

$$\alpha_1 = 2 - \alpha_2 \Rightarrow 4(2 - \alpha_2) + 2\alpha_2 = 1$$

$$\Rightarrow 8 - 4\alpha_2 + 2\alpha_2 = 1 \rightarrow -2\alpha_2 = -7 \Rightarrow \boxed{\alpha_2 = \frac{7}{2}}$$

$$\boxed{\alpha_1 = 2 - \alpha_2 = 2 - \frac{7}{2} = -\frac{3}{2}}$$