Quiz 7: November 9, 2021

Name: Solution key

Student ID: ______

1. (5 points) If $n > 1$ and $n \in$ the Integers, prove by Induction that

$$3^n \geq 2n + 5$$

Clearly identify your Basis Case (1 points), your Inductive Step (3 points), and your Inductive Hypothesis (1 points).

2. (5 points) If $n > 0$ and $n \in$ the Integers, prove by Induction that

$$2^n \leq 2^{n+1} - 2^{n-1} - 1$$

Clearly identify your Basis Case (1 points), your Inductive Step (3 points), and your Inductive Hypothesis (1 points).

Q1

$$\boxed{3^n \geq 2n+5} \quad P(n)$$

Basis case: $n=2$

$3^2 = 9 \geq 2(2)+5 = 9$

Inductive Hypothesis:

$3^n \geq 2n+5$

Inductive Step:

$$\boxed{3^{n+1} \geq 2(n+1)+5 = 2n+7} \quad P(n+1)$$

$3^{n+1} = 3^n \cdot 3 \geq (2n+5) \cdot 3 = 2n+15 \geq 2n+7 = 2(n+1)+5$

Q2

$$\boxed{2^n \leq 2^{n+1} - 2^{n-1} - 1} \quad P(n)$$

Basis case: $n=1$

$2^1 = 2 \leq 2^{1+1} - 2^{1-1} - 1 = 2^2 - 2^0 - 1 = 4-1-1 = 2$

$2 \leq 2$

Inductive hypothesis

$2^n \leq 2^{n+1} - 2^{n-1} - 1$

Inductive Step

$\boxed{2^{n+1} \leq 2^{n+2} - 2^n - 1}$ $P(n+1)$

$$2^{n+1} = 2 \cdot 2^n \leq 2 \cdot \left(2^{n+1} - 2^{n-1} - 1\right) = 2^{n+2} - 2^n - 2 \leq 2^{n+2} - 2^n - 1$$