

Quiz 6: November 4, 2021

Name: _____

Student ID: _____

To remind you of the Brain-dead recipe for Inductive Steps:

1. Write down the LHS of $P(n+1)$
2. Rewrite LHS of $P(n+1)$ to include LHS $P(n)$
3. Replace LHS of $P(n)$ with RHS $P(n)$
4. Rewrite so RHS becomes RHS $P(n+1)$

1. (10 points) If $n \in$ the Integers, and $n \geq 0$, prove by Induction that

$$\sum_{k=0}^n i \cdot i! = (n+1)! - 1.$$

Clearly identify your Basis Case (2 points), your Inductive Step (6 points), and your Inductive Hypothesis (2 points).

Base Case: $k=0 \rightarrow 0 \cdot 0! = 0 = (0+1)! - 1 = 1 - 1 = 0$

Inductive Hypothesis: assume $P(n)$ is true for all $n \geq 0$:

I.H $P(n) = \sum_{k=0}^n i \cdot i! = (n+1)! - 1$

Prove: $\sum_{k=0}^{n+1} i \cdot i! = (n+2)! - 1$

LHS: $P(n+1) = \sum_{k=0}^{n+1} i \cdot i! = \underbrace{\sum_{k=0}^n i \cdot i!}_{\text{by I.H}} + (n+1)(n+1)! =$

$= (n+1)! - 1 + (n+1)(n+1)! = (n+1)! (n+1+1) - 1$
 $= (n+1)! (n+2) - 1 = (n+2)! - 1 = \text{R.H.S.} \checkmark$