Quiz 5: November 4, 2021

Name: ____

Student ID: ____

For this quiz, refer to these helpful tables for Propositional Calculus:

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

$p \to q \equiv \neg p \vee q$	Relation by Implication (RBI)
$p \to q \equiv \neg q \to \neg p$	Contraposition
$p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	Definition of Biconditional
$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$	Alternate Definition of xor

Solution

$\binom{33}{13} \cdot 2^{20}$ is coefficient.

1. (2 points) What is the coefficient of $x^{13}y^{20}$ in the expansion of $(x-2y)^{33}$

$$\binom{33}{13} x^{13} (-2y)^{20} = \boxed{\binom{33}{13}(-2)^{20}} \cdot \underline{x^{13} \cdot y^{20}}$$

2. (2 points) Prove that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ algebraically.

From Right to Left:

$$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!\,(n-k)!} + \frac{n!}{(k-1)!\,(n-k+1)!}$$

$$= \frac{n! \cdot (n+1-k)}{k!\,(n+1-k)!} + \frac{n! \cdot k}{k!\,(n+1-k)!} = \frac{n!\,(n+1)}{k!\,(n+1-k)!} = \binom{n+1}{k} \quad \#$$

3. (2 points) Explain why $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ by a combinatorial argument. If you don't know what a combinatorial argument is, it is english plus picture plus handwaving. We can't answer any more questions about it.

→ In a $\underline{(n+1)\text{-size}}$ set: $\{x_1, x_2, x_3, \dots, x_i, \dots, x_{n+1}\}$

→ x_i is red. Others are black.

→ If I choose k elements from this set.

4. (4 points) Use a formal proof using rules of inference to prove that given Hypotheses $(p \wedge q \to r)$, $(q \to p)$, and q, you can prove r.

→ There only are 2 results:

$\begin{cases} x_i \text{ in your selected } k\text{-size set. } ① \\ x_i \text{ is not in your selected set. } ② \end{cases}$

→ For ①: you need to select other $(k-1)$ elements from left n elements

For ②: you need to select other k elements from the n left elements

→ #①: $\binom{n}{k-1}$ → #②: $\binom{n}{k}$

$\#$

1. (2 points) What is the coefficient of $x^{13}y^{20}$ in the expansion of $(x-2y)^{33}$

2. (2 points) Prove that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ algebraically.

3. (2 points) Explain why $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ by a combinatorial argument. If you don't know what a combinatorial argument is, it is english plus picture plus handwaving. We can't answer any more questions about it.

4. (4 points) Use a formal proof using rules of inference to prove that given Hypotheses $(p \wedge q \to r)$, $(q \to p)$, and q, you can prove r.

Problem 4.

1.	$(p \wedge q) \to r$	H_1
2.	$q \to p$	H_2
3.	q	H_3
4.	p	Mp 2, 3
5.	$p \wedge q$	Conjunction 3, 4
6.	r	Mp 1, 4