Quiz 5: November 4, 2021

Name: _____

Student ID: _____

For this quiz, refer to these helpful tables for Propositional Calculus:

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

$p \rightarrow q \equiv \neg p \vee q$	Relation by Implication (RBI)
$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Contraposition
$p \Leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Definition of Biconditional
$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$	Alternate Definition of xor

1. (2 points) What is the coefficient of $x^{13}y^{20}$ in the expansion of $(x-2y)^{33}$

2. (2 points) Prove that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ algebraically.

3. (2 points) Explain why $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ by a combinatorial argument. If you don't know what a combinatorial argument is, it is english plus picture plus handwaving. We can't answer any more questions about it.

4. (4 points) Use a formal proof using rules of inference to prove that given Hypotheses $(p \wedge q \rightarrow r), (q \rightarrow p)$, and q, you can prove r.