

Quiz 4 Solutions

1. How many numbers between 1 and 888,881 are divisible by 2,5, **or** 13?

$$x = \left\lfloor \frac{888,881}{2} \right\rfloor + \left\lfloor \frac{888,881}{5} \right\rfloor + \left\lfloor \frac{888,881}{13} \right\rfloor - \left\lfloor \frac{888,881}{\text{lcm}(2,5)} \right\rfloor - \left\lfloor \frac{888,881}{\text{lcm}(2,13)} \right\rfloor - \left\lfloor \frac{888,881}{\text{lcm}(5,13)} \right\rfloor + \left\lfloor \frac{888,881}{\text{lcm}(2,5,13)} \right\rfloor$$

Answer for typo: How many numbers between 1 and 888,881 are divisible by 2,5, **and** 13?

$$x = \left\lfloor \frac{888,881}{2 * 5 * 13} \right\rfloor$$

2. How many numbers between 1 and 888,881 are **not** divisible by 2,5, and/or 13?

$$888,881 - x$$

3. Give the number of Combinations of n things taken k at a time ($C(n,k)$) in terms of Permutations, and explain why this makes sense in English.

$$C(n, k) = \frac{P(n, k)}{P(k, k)} = \frac{P(n, k)}{k!} = \frac{n!}{k!(n - k!)}$$

With permutations we care about the order of elements and with combinations we don't. If we are interested in knowing the number of combinations of n things taken k at a time, we can arrange all n things in some arbitrary order and pick k of them. The number of ways we can do this is the number of permutations of n things taken k at a time. However, since with combinations we are not interested in the order of the k things, we need to remove the permutations that represent the same combinations. There are $k!$ such permutations. By dividing by $k!$ we remove duplicates when counting the number of combinations.

In short, the number of combinations is $k!$ times smaller than the number of permutations.

4.

- a) Give the formula for how many 5-card hands have 3 of a kind (but not 4 of a kind, and not 3 of a kind and 2 of a kind).

$$\binom{13}{1} * \binom{4}{3} * \binom{12}{2} * \binom{4}{1} * \binom{4}{1} = 54,912$$

- b) Give the formula for how many 5-card hands have 2 pairs and a fifth card that is a different rank than either of the pairs.

$$\binom{13}{2} * \binom{4}{2} * \binom{4}{2} * \binom{11}{1} * \binom{4}{1} = 123,552$$

c) Explain why 3 of a kind is a more valuable hand than 2 pairs, in English.

There are more 5-card hands that have 2 pairs than 3 of a kind, therefore, the probability of getting 3 of a kind is smaller, making 3 of a kind more valuable.