Quiz 3: October 14, 2021

Name: ______________________________

Student ID: ______________________________

For this quiz, refer to these laws and identities for Propositional Calculus:

Name	Equivalence
Identity Laws	$p \wedge \mathbf{T} \equiv p$ $\quad$ $p \vee \mathbf{F} \equiv p$
Domination Laws	$p \vee \mathbf{T} \equiv \mathbf{T}$ $\quad$ $p \wedge \mathbf{F} \equiv \mathbf{F}$
Idempotent Laws	$p \vee p \equiv p$ $\quad$ $p \wedge p \equiv p$
Double Negative Law	$\neg(\neg p) \equiv p$
Commutative Laws	$p \vee q \equiv q \vee p$ $\quad$ $p \wedge q \equiv q \wedge p$
Associative Laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $\quad$ $(p \wedge q) \wedge r \equiv q \wedge (p \wedge r)$
Distributive Laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $\quad$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\quad$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Absorption Laws	$p \vee (p \wedge q) \equiv p$ $\quad$ $p \wedge (p \vee q) \equiv p$
Negation Laws	$p \vee \neg p \equiv \mathbf{T}$ $\quad$ $p \wedge \neg p \equiv \mathbf{F}$
Def. of implication	$(p \rightarrow q) \equiv (\neg p \vee q)$
Def. of equivalence	$\mathbf{p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)}$

Solution:

1. The questions below start with the formula $f(a, b, c) = (\neg a \wedge \neg b) \vee (a \wedge c)$

   (a) (1 points) Give us the truth table for $f(a, b, c)$.

   (b) (2 points) Give us the Conjunctive Normal Form (CNF–also known as Product of Sums, POS) for $f(a, b, c)$

   (c) (2 points) Give the Dual of your solution for Part b of this question.

2. (5 points) Use the provided laws and identities to prove that $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.

1a.

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

1b.

CNF: $(a \vee \neg b \vee c) \wedge (a \vee \neg b \vee \neg c) \wedge (\neg a \vee b \vee c) \wedge (\neg a \vee \neg b \vee c)$

1c.

Dual: $(a \wedge \neg b \wedge c) \vee (a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge b \wedge c) \vee (\neg a \wedge \neg b \wedge c)$

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   (c) (2 points) Give the Dual of your solution for Part b of this question.

2. (5 points) Use the provided laws and identities to prove that $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.

$[p \wedge (\neg p \vee q)] \rightarrow q$	def. of Implication
$[(p \wedge \neg p) \vee (p \wedge q)] \rightarrow q$	Distributive Law
$[\ 0 \vee (p \wedge q)] \rightarrow q$	Negation Law
$p \wedge q \rightarrow q$	Identity Law
$\neg (p \wedge q) \vee q$	Def. of Implication
$\neg p \vee (\neg q \vee q)$	De Morgan's Law
$\neg p \vee T$	negation Law
T	