Quiz 10: December 2, 2021

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1. (5 points) if $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, with $a_0 = 4, a_1 = 1$, find the closed form of this Linear Homogeneous Recurrence Relation of order 2 with constant coefficients.

   Make sure you give your characteristic equation (1 points), find your roots (1 points), write down the solution in terms of unknown constants (1 points), and then solve the two equations with two unknowns to come up with a final solution (2 points).

$A_n = 2A_{n-1} - A_{n-2}$ $A_0 = 4$ $A_1 = 1$

CE: $r^2 - 2r + 1 = 0$

$(r-1)(r-1) = 0$ $r_1 = 1, r_2 = 1$

$A_n = \alpha_1 r_1^n + \alpha_2 n r_2^n$

$A_n = \alpha_1 + \alpha_2 n$

$A_0 = 4 = \alpha_1 + \alpha_2 \cdot 0$

$= 4 = \alpha_1$

$A_1 = 1 = 4 + \alpha_2$

$\alpha_2 = -3$

$A_n = 4 - 3n$

2. (5 points) Prove the above closed form correct via proof by induction. Clearly identify your Basis Cases (2 points), your Inductive Step (2 points), and your Inductive Hypothesis (1 points).

$A_n = 2A_{n-1} - A_{n-2}$ $A_0 = 4$ $A_1 = 1$

$A_n = 4 - 3n$

Base

$n = 0$

$A_0 = 4 = 4 - 3 \cdot 0$ ✓

$n = 1$

$A_1 = 1 = 4 - 3 = 1$ ✓

I.S.

Assume: $A_k = 4 - 3k$ for all $0 \leq k \leq n$

Prove: $A_{n+1} = 4 - 3(n+1) = 4 - 3n - 3 = 1 - 3n$

$A_{n+1} = 2A_n - A_{n-1}$

$= 2(4 - 3n) - (4 - 3(n-1))$

$= 8 - 6n - (4 - 3n + 3)$

$= 8 - 6n - 4 + 3n - 3$

$= 1 - 3n$ ✓